ECONOMICS
& Sociology

Dick, T., Čermáková, K., Rotschedl, J., & Popescu, I.A. (2026). Compliance risk and pension sustainability: A policy frontier approach. *Economics and Sociology*, *19*(1), 61-91. doi:10.14254/2071-789X.2026/19-1/4

COMPLIANCE RISK AND PENSION SUSTAINABILITY: A POLICY FRONTIER APPROACH

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Received: June, 2025
1st Revision: January, 2026
Accepted: March, 2026

DOI: 10.14254/2071-789X.2026/19-1/4

ABSTRACT. This paper analyses the fiscal and structural implications of a "compliance shock" - a sudden decline in social security contributions - within pay-as-you-go (PAYG) pension systems. We characterise this phenomenon as an immediate budget shock rather than a demographic one, demonstrating that because the ratio of contributors to retirees remains fixed on impact, benefits must fall immediately unless policy intervenes. Our findings reveal that demographic instruments, such as raising the retirement age, are insufficient for short-term stabilisation because they only gradually alter the balance only gradually. Consequently, maintaining benefit stability requires a front-loaded response through contribution rates. We identify a precise policy trade-off frontier between higher contributions and tighter retirement criteria. Under realistic adjustment costs, we show that the optimal long-term response is a hybrid policy mix rather than an extreme reliance on a single instrument. However, statutory contribution caps can render benefit protection infeasible, making retirement-age increases a necessity rather than a choice, especially as future demographic ageing intensifies. Our calibration suggests that these shocks generate economically significant pressures, with the timing and sequencing of reforms determined by the relative costs of each policy tool. Regardless of the specific pension philosophy (Bismarckian or Beveridgean), the underlying fiscal mechanics remain consistent. We conclude that pension systems should internalise compliance risk by adopting automatic balancing mechanisms. By replacing ad hoc crisis management with predictable state-contingent adjustments, policymakers can better stabilise expectations and ensure long-term institutional resilience.

JEL Classification: J24, H55, H26, D72

Keywords: PAYG pension system, compliance shock, dependency ratio, replacement rate, retirement-age policy, fiscal sustainability, policy frontier, institutional credibility

Introduction

Pay-as-you-go (PAYG) pension systems have been at the centre of intense academic and political debates for decades, and the dominant argument is almost always the same: the pension problem is, in essence, a demographic problem. Population ageing, declining fertility, increasing life expectancy, these are the variables that, in the classic tradition of overlapping generational models initiated by Samuelson (1958), determine the pressure on contribution rates, replacement rates, and the long-term solvency of public pension systems. Given the positive effect of social expenditure on economic growth through the multiplier effect (Yurchyk et al., 2024), the problem of pension system structure, funding and perception by contributors remains a pressing and unresolved issue (Brycz et al., 2024, Bednarczyk et al., 2023). In this paradigm, the available public policy instruments are well known: adjustment of retirement age, changing contribution rates, and benefit indexation rules (Ayuso et al., 2021; Dankiewicz et al., 2022). The debate is not whether these instruments are necessary but in what proportion and in what sequence they should be used.

However, this demographic paradigm leaves out a distinct and potentially equally severe source of fiscal fragility: deteriorating contributory compliance. A PAYG system can enter into acute financial distress not only when the number of retirees increases relative to that of contributors, but also when a significant fraction of the statutory contribution base is no longer translated into actual revenue collected. Contribution evasion, underreporting of income, legal exceptions, and administrative deficiencies create a gap between what should be collected and what is actually collected, a gap that immediately affects the pension system budget, without changing, in the short term, the ratio of retirees to contributors. This distinction is analytical, but it has direct practical implications: While demographic pressure builds up slowly and can be anticipated, a compliance shock can strike suddenly and require an immediate public policy response, for which demographic instruments, by their nature gradual, are ill-suited.

This article starts from this observation and builds a formal model of the fiscal mechanics of a compliance shock in a pure PAYG system. The central contribution is not to reaffirm that compliance matters; the existing literature has already documented this relationship (McGillivray, 2001; Li et al., 2020), but to formalise the compliance shock as a distinct fiscal disturbance, with its own logic, separable from demographic dynamics. The model shows that a permanent drop in compliance immediately and proportionally shifts the budget curve of the pension system, while the demographic dependency ratio remains unaffected in the short run. Several model-based results follow from this premise: the immediate effect of a compliance shock is a budgetary shock, not a demographic one; retirement age policy cannot neutralise the shock on impact, because the dependency ratio is a state variable that cannot jump; any immediate stabilisation of the replacement rate requires an immediate response by the contribution rate; and the feasible long-term combinations of contribution increases and retirement criteria can be summarised by a precise public policy frontier.

The analysis is deliberately institutionally neutral with respect to the classical Bismarck-Beveridge distinction. The framework is built around the PAYG budgetary identity and a reduced-form compliance margin that captures the gap between statutory and actual collections; these objects appear equally in Bismarckian schemes based on earnings-related contributions and in Beveridgean ones with flat-rate or minimum pensions. What differs between the regimes is not the accounting core, but the intensity and interpretation of the compliance channels: In more Bismarckian systems, compliance incentives are shaped by the perceived contribution-benefit link, while in more Beveridgean systems, they are more closely connected to considerations of redistribution and legitimacy. Therefore, the model should be read as a parsimonious representation of PAYG financing under imperfect remittance conditions, applicable to both institutional archetypes, not as an analysis of a specific pension philosophy.

1. Literature review

The literature on pay-as-you-go (PAYG) pension systems has traditionally focused on demographic sustainability, especially the interaction between fertility, longevity, labour-force participation, and the retiree-to-contributor ratio. In the classic overlapping-generations tradition, PAYG is understood as an intergenerational transfer mechanism whose implicit return is tied to population and productivity growth (Samuelson, 1958). Building on this foundation, subsequent work has examined how ageing, declining fertility, and increased life expectancy place persistent pressure on contribution rates, replacement rates, and pension system solvency. In this broader sustainability debate, the main policy instruments have usually been identified as contribution rates, retirement ages, and benefit indexation rules (Ayuso et al., 2021; Samuelson, 1958; Streimikiene, 2025).

This traditional approach neglects the stress on the pension system caused by the slow-moving demographic change. The present model isolates a distinct source of fiscal imbalance: a fall in effective remittances relative to statutory contribution obligations. In that sense, the paper speaks to a more specific strand of research dealing with contribution evasion, under-reporting, and imperfect remittance. That literature has long shown that PAYG systems may become financially fragile even when the statutory architecture remains unchanged because legal contribution rates do not necessarily translate into actual revenue collection (McGillivray, 2001). We believe that this distinction is analytically important: demographic pressure changes the dependency ratio gradually, whereas compliance deterioration affects pension-system revenue immediately. The compliance shock is central to the analytical model presented in this paper. We explicitly treat compliance as a wedge between statutory and effective collections, while leaving demographic dynamics governed by a separate state equation.

In this research, we follow up our conceptual foundation of the broader framework of PAYG system sustainability. Rotschedl (2015) argued that the sustainability of PAYG pension systems cannot be reduced to demographic ageing alone, but must also account for income structure, labour market conditions, and other fiscal determinants that affect the effective contribution base. Rotschedl et al. (2024) further extended this line of reasoning through a comparative analysis of PAYG schemes over the period 1993–2023, showing that the sustainability of the pension system is shaped not only by demographic parameters, but also by institutional settings, family policy, and broader macroeconomic conditions.

A first key branch of related literature concerns the salience channel. When contribution burdens become more visible, or when contribution rates increase sharply, compliance may weaken. Blakemore et al. (1996) showed in the context of payroll-tax enforcement that higher contribution-like tax burdens can induce greater noncompliance. Iturbe-Ormaetxe (2015) similarly demonstrated that the division and visibility of social security contributions can affect labour market outcomes, suggesting that the behavioural consequences of payroll taxation depend not only on levels but also on how contribution burdens are perceived. Li et al. (2020) further found that missing social security contributions are systematically related to contribution rates and to the broader corporate tax environment. This literature supports the first mechanism in the present model: a revenue-side shock may emerge not only from weak enforcement, but also from the behavioral response to salient contribution increases.

The second strand concerns the credibility channel. In PAYG systems, current contributors must believe that today's payments will be translated into future entitlements in a sufficiently reliable and predictable way. Roubal (2023) suggests that the transition to digital commerce and the "horizontal transformation" of markets during the COVID-19 pandemic have fundamentally altered how individuals perceive stability and freedom. If a pension system appears opaque or its future benefits uncertain, contributors may experience what Roubal describes as "choice overload," leading to passivity or a psychological resignation from the social contract (Roubal, 2023). The similar biases are highlighted by Brycz & Brycz (2025). Besley et al. (2005) argued that pension promises are only sustainable if governance arrangements make them credible. Kangas et al. (2022), in an experimental survey on the Finnish pension reform, showed that perceived legitimacy and acceptance improve when participants better understand the reform and regard it as credible and fair. Related evidence from the Netherlands indicates that trust in pension providers is sensitive to crises and reforms; van Dalen et al. (2022) found that trust in pension institutions varies with reform episodes and financial conditions, while Mangan et al. (2025) linked trust shocks to additional private pension-saving behaviour and Dick (2025) shows that the predicted probability of third-pillar pension participation varies significantly among birth cohorts, which may be driven by credibility of the statutory pension system. Together, this literature strongly supports the assumption that compliance is not merely administrative but is partly endogenous to the credibility of the contribution–benefit contract.

The third branch is the political instability channel. PAYG pension promises are inherently exposed to future political decisions because the system is not funded in advance, but depends on the continuing willingness of governments to honour and update benefit rules. McHale (2001) documented that social security benefit rules are repeatedly altered through the political process and that this creates genuine political risk for contributors. Shoven et al. (2006) similarly framed PAYG pensions as subject to political risk, not only market or demographic risk. The concern is not merely that reforms occur, but that they may occur in a sequence of ad hoc reversals, shifting burdens across cohorts in a way that makes the long-run social contract appear unstable. In the notes of the literature, this mechanism is explicitly linked to the reduced-form treatment of compliance shocks of the present model, where policy volatility itself reduces the effective remitted share of the contribution base.

A fourth and closely related branch concerns distributional fairness and the contribution–benefit link. If contributors perceive the system as too weakly linked to their own contributions, or as unfairly redistributive across cohorts or income groups, compliance may fall. Kumler et al. (2020) provided evidence from Mexico that stronger information and incentives related to payroll tax reporting can improve compliance. Leroux et al. (2019) developed a political-economy framework in which imperfect tax compliance interacts with contributive pension design, showing that a tighter contribution–benefit link can strengthen truthful reporting and broaden the effective tax base. Choi (2009), in an OECD comparative review of pension schemes for the self-employed, also highlighted the persistent compliance problems that arise when contribution obligations are weakly enforced or poorly aligned with perceived benefits. This literature supports the fairness channel and also helps explain why the results are applicable to both Bismarckian and Beveridgean PAYG systems, although the intensity of the compliance mechanism differs between institutional settings.

Within this background, the main contribution of the present paper is not simply to restate that compliance matters, but to formalise it as a distinct fiscal shock within a PAYG framework, following a broader tradition of formal modelling of complex socioeconomic systems (Lansky et al., 2022). This paper shows that a permanent fall in compliance shifts the pension budget set downward one for one, while demography remains unchanged on impact. From this, several model-based propositions follow: first, the immediate effect of a compliance shock is a budget shock, not a demographic shock; second, retirement age policy cannot neutralize the shock on impact because the dependency ratio is a state variable; third, any immediate stabilisation of the replacement rate requires an instantaneous contribution response; and fourth, the long-run feasible combinations of contribution increases and retirement-age tightening can be summarised by a precise policy frontier. This makes the paper a bridge between the literature on PAYG sustainability, the literature on contribution evasion, and the literature on institutional credibility and pension policy design.

2. The baseline model

2.1. Environment

Time is discrete, $t = 0,1,2,\dots,T$. The pension system is pure PAYG, all contributions collected in t are paid as pensions in t thus there is no capital accumulation within the pension system. We normalize productivity and prices so that productivity $A \equiv 1$ and inflation $\pi \equiv 0$, nominal issues never enter. Households are in one of two states, (1) being a contributor N_t to the PAYG or (2) being a receiver R_t of pension benefits from the pension system. The compliance rate $c_t \in (0,1]$ is the effective fraction of the statutory contribution base that is remitted to the PAYG system at date t. Interpreted broadly, c_t aggregates all forces that shrink the contributory base conditional on being in the contributor pool like non-remittance and evasion, underreporting of earnings, legal exemptions, caps and administrative shortfalls. We do not track individual histories, so current non-compliance does not remove someone's future eligibility. Formally, if the statutory base is $s_t\omega_t N_t$ with $s_t \in [0,1]$ being the statutory contribution rate and $\omega_t > 0$ the wage, observed collections are

$$Y_t = s_t\omega_t c_t N_t,$$

so that

$$c_t = \frac{Y_t}{s_t\omega_t N_t}.$$

Perfect enforcement and full coverage imply $c_t = 1$ therfore any wedge between statutory and actual collections appears as $1 - c_t$.

We interpret the compliance rate $c_t \in (0,1]$ as the share of the contributory base that actually remits payroll taxes at time t. Beyond political cycles, this rate is implicitly tied to wage inflation at the potential of the economy. As Kaderabkova et al. (2019) show, misalignments in unit wage costs can signal underlying instability in the national economy. In the context of our model, such macro-mezzo level imbalances may manifest as administrative shortfalls or increased legal exemptions as firms struggle with rising real unit wage costs, thereby reducing the effective fraction of the statutory base that reaches the pension system.

Beyond macro conditions, c_t is sensitive to policy stability and perceived fairness of PAYG rules. The broader environment of globalization, as reviewed by Neugebauer et al. (2024), suggests that national pension policies no longer operate in isolation; rather, they are subject to "globalization-induced" volatility that can undermine the perceived credibility of the social contract. As Jasova et al. (2022) argue in the context of the first pandemic wave, the effectiveness of government measures (or the lack thereof) creates a fundamental tension between economic performance and social stability. In our model, this tension is captured by the compliance rate; if the government fails to provide adequate institutional 'shielding', the perceived credibility of the social contract may erode, triggering the compliance shocks analyzed in the following chapters. Electoral cycles can therefore trigger compliance shocks through four mechanisms:

1. *Salience channel* $(-\eta\Delta s_t)$*:* Short-horizon governments often rely on the contribution instrument s_t to avoid politically costly age reforms. Sudden hikes in s_t increase salience of the payroll wedge and trigger avoidance or evasion.

2. *Credibility channel* $(-\zeta\mathbb{I}\{\text{policy reversal at } t\})$*:* Pre-/post-election shifts like temporary contribution holidays, one-off bonuses, ad-hoc indexation degrade expectations that today's contributions buy tomorrow's benefits, eroding trust.

3. *Instability channel* $(-\theta Var_\kappa)$*:* Elevated short-run volatility of the contribution rate around elections signals an unreliable contribution-benefit contract, even holding the level of the contribution rate fixed, instability reduces voluntary compliance.

4. *Distributional fairness channel* $(-\gamma(s_t - s_t^{rule}))$: When the actual statutory payroll rate exceeds the announced rule needed to sustain the replacement rate, the burden is shifted toward current workers – especially younger cohorts – depressing compliance.

A parsimonious reduced form that captures these channels is:

$$c_{t+1} = \bar{c} - \eta\Delta s_t - \theta Var_\kappa(s_{t-\kappa:t}) - \zeta\mathbb{I}\{\text{policy reversal at } t\} - \gamma(s_t - s_t^{rule}) + \varepsilon_{t+1}, \quad (PC)$$

with $\eta, \theta, \zeta, \gamma > 0$ as marginal effects of compliance with respect to (i) a one-period social security tax change, (ii) reversal event, (iii) recent policy instability and (iv) intergenerational fairness. Here Var_κ is a κ-period rolling variance that proxies policy instability around elections. The reversal indicator $\mathbb{I}\{.\} \in \{0,1\}$ equals one when the government deviates sharply from its previously announced path, zero if not. $\bar{c}$ is the baseline compliance under stable policy while $\Delta s_t = s_t - s_{t-1}$ indicates salient changes in the rate and s_t^{rule} represents the optimal policy as described in chapter 2. The residual ε_{t+1} has a mean of zero and is serially uncorrelated capturing factors outside current policy like business-cycle effects. Equation (PC) parsimoniously maps electoral volatility - salient tax hikes, policy instability, credibility breaks, and rule-deviation unfairness - into next-period compliance, thereby linking political cycles to the compliance shocks analyzed in the following chapters.

With a contributor base of $N_t > 0$ and $R_t > 0$ as a retiree base, the dependency ratio is noted as retirees per contributor, the single state variable is defined as

$$d_t \equiv \frac{R_t}{N_t}.$$

Switching from state of a contributor to a retiree between the course from t to $t+1$ is captured by the retirement inflow rate $f_t \in (0,1)$ while $m_t \in (0,1)$ is denoted as the retiree exit rate. The contributor base increases with the growth rate of $n \geq 0$ due to higher fertility rate, migration or higher labour force participation.

We impose $1 + n - f_t > 0$ to ensure that next period's contributor base must remain positive, and typically $0 < f_t < n + m$ such that a steady state exists.

Overall compliance c_t and the contribution rate s_t determine how much revenue is collected from each contributor, the retirement inflow rate f_t and exit rate m determine how demography evolves. Only f_t and exogenous n and m can move the value of the dependency ratio d_t while s_t and c_t do not affect d_t directly.

2.2. PAYG accounting

Collected contributions in period t are

$$Y_t = s_t \omega_t c_t N_t. \quad (1)$$

With per-retiree pension of $b_t = Y_t / R_t$, the replacement rate is

$$\rho_t \equiv \frac{b_t}{\omega_t} = \frac{s_t c_t}{d_t}. \quad (2)$$

Equation (2) represents the budget hyperbola $\rho_t d_t = s_t c_t$. It pins down a per-period trade-off between generosity ρ_t and demography d_t given the income generating instruments $s_t$and c_t. The real wage ω_t cancels out in ρ_t but scales the level of benefits $b_t = \rho_t \omega_t$.

2.3. Demography and one state law of motion

The numbers of contributors and retirees satisfy

$$N_{t+1} = (1 + n - f_t)N_t \quad (3)$$

$$R_{t+1} = (1 - m)R_t + f_t N_t. \quad (4)$$

Let n denote net contributor growth, $m \in (0,1)$ the retiree exit rate and $f_t \in [0, n + m)$ the retirement inflow intensity. Combining (3) and (4) yield a single-state-transition for the dependency ratio, the one-state law of motion:

$$d_{t+1} = D(d_t; f_t) = \frac{R_{t+1}}{N_{t+1}} = \frac{(1 - m)d_t + f_t}{1 + n - f_t}, \qquad f_t \in [0,1 + n), n > -1. \quad (5)$$

For $\partial d_{t+1}/\partial d_t = (1 - m)/(1 + n - f_t) \in (0,1)$ with $0 < f_t < n + m$ the next period's burden rises with today's burden but less than one-for-one, a force toward mean reversion. c_t and s_t do not enter the law of motion since compliance and contribution do not change demography contemporaneously.

With $d_t = R_t/N_t$, the dependency ratio evolves according to the one-state map $D(d_t; f_t)$ which has a unique fixed point $d^*(f) = \frac{f}{n+m-f}$ and is a global contraction in d with gain

$$D_d = \frac{\partial D}{\partial d} = \frac{1 - m}{1 + n - f} \in (0,1).$$

2.4. Steady state, global stability, and speed of convergence

With constant primitives (n, m, c) and fixed policies (s', f'), the fixed point of (5) solves

$$d^* = \frac{f}{n + m - f}, \qquad 0 < f < n + m. \quad (6)$$

A stationary dependency ratio requires inflow and outflow to balance. Hence d^* is the ratio of the retiree inflow per contributor to the net drain that pushes the system back toward balance. The viability condition $f < n + m$ is economically transparent: the system cannot stabilize if the retirement inflow exceeds the combined force of contributor growth and retiree exit.

Write (5) as $d_{t+1} = ad_t + b$ with

$$a \equiv \frac{1 - m}{1 + n - f} \in (0,1), \quad b \equiv \frac{f}{1 + n - f},$$

then

$$d_t = a^t d_0 + (1 - a^t)d^*, \qquad |d_t - d^*| = a^t|d_0 - d^*|. \quad (7)$$

Equation (7) says the dependency ratio is a convex combination of its initial condition d_0 and the steady state d^* with a time-varying weight a^t on d_0 that decays geometrically.

Because the slope $a \in (0,1)$, the map is a contraction. From any $d_0 > 0$, the path converges monotonically to d^* with no oscillations. Unanticipated one off shocks (e.g., a compliance shock to c) may move the system away from the previous steady state, but the new dynamics remain well-behaved and converge to the new d^*.

Define the half-life $t_{1/2}$ as the time it takes for the gap $|d_t - d^*|$ to fall by 50%. Since $|d_{t+k} - d^*| = a^k|d_t - d^*|$ we obtain

$$t_{1/2} = \frac{\ln(1/2)}{\ln a} > 0.$$

$t_{1/2}$ summarizes how quickly a reform of f translates into a visible reduction in the demographic burden. Since $\partial t_{1/2}/\partial f > 0$, a credible, sustained reduction in f reduces both the level d^* and the half-life, strengthening the medium-term payoff of pensionable-age policy. Thus, a tighter retirement-entry policy unambiguously shortens the half-life and accelerates the return of d_t to d^*, a milder reform lengthens it.

From (2) and (6) the steady state replacement rate can be derived

$$\rho^* = \frac{sc}{d^*} = sc\frac{n+m-f}{f}. \quad (8)$$

The steady-state generosity (8) shows, that in the long run, the replacement rate equals the effective contribution capacity per contributor sc, multiplied by the demographic leverage factor $(n+m-f)/f$. This factor is the ratio of outflows easing the dependency to the inflow into retirement. The PAYG system trades today's payroll base against the retiree burden. In steady-state, demography pulls constantly at the budget: higher retiree inflow f raises the number of beneficiaries relative to contributors, while higher contributor growth n and retiree exit m push the other way.

3. Permanent compliance shock (c↓)

3.1. Mechanics without policy intervention

In each period the PAYG budget satisfies

$$\rho_t d_t = s_t c_t,$$

with contribution rate $s_t \geq 0$, compliance $c_t \in (0,1]$, dependency ratio $d_t > 0$, and replacement rate $\rho_t \geq 0$. Consider a discrete, permanent compliance shock at date τ, $c_{\tau^-} = c_0 \rightarrow c_{\tau^+} = c_1 < c_0$. Holding policy and demography fixed on impact $s_\tau = s_0, d_\tau = d_\tau^-$, the replacement rate drops multiplicatively

$$\rho_{\tau^+} = \frac{s_0 c_1}{d_\tau} = \frac{c_1}{c_0}\rho_{\tau^-} \quad (9)$$

and remains at the new level thereafter if no policy intervention occurs. Equivalently, $\rho(sc,d) = sc/d$ implies the unit elasticities $\varepsilon_{\rho,c} = 1$ and $\varepsilon_{\rho,d} = -1$, such that a 1% loss in compliance reduces generosity by 1% at fixed s and d. The per-person normalization in the model makes the one-for-one relationship visible.

While d_t follows (5) unchanged and converges to the same d^* in (6). Hence

$$\rho_{new}^* = \frac{sc_1}{d^*} = \frac{c_1}{c_0}\rho_{old}^* = \frac{c_1}{c_0}\frac{s_0 c_0}{d_t}.$$

A permanent compliance loss is a pure budget shock, shifting the budget hyperbola (2) downwards. The entire future path of the replacement rate falls proportionally on impact with the factor c_1/c_0 in the long run. The law of motion (5) does not contain the compliance rate. Hence the path and speed of the replacement rate are exactly as before the shock converging monotonically towards (6).

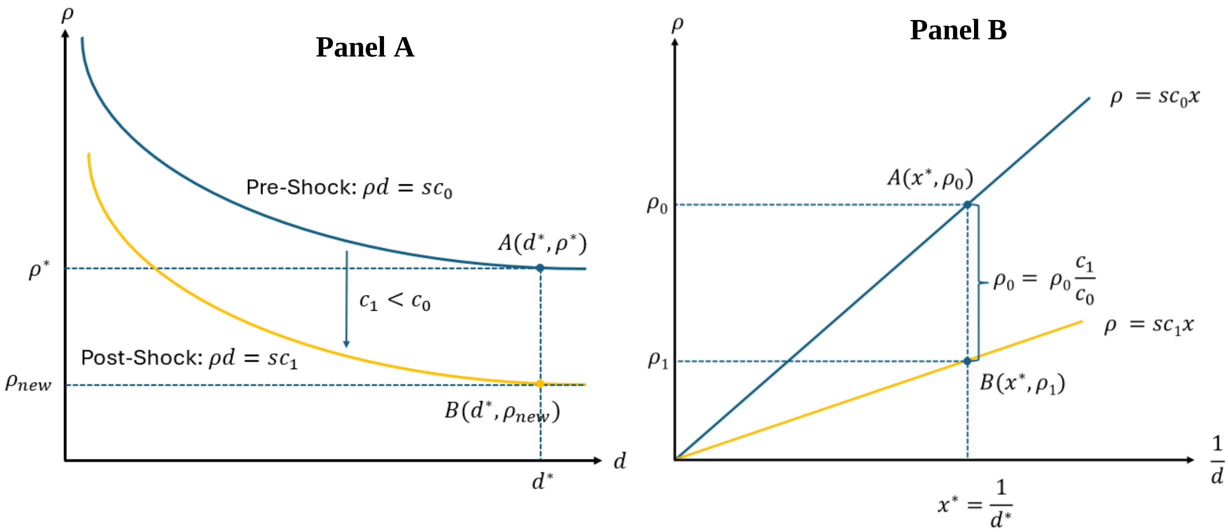

Figure 1. Budget hyperbola (Panel A) and linear form (Panel B) of the PAYG identity

For a given contribution and compliance rate s and c, the period budget is determined by equation (8). The budget hyperbola captures all feasible d and ρ combinations for which the budget equation holds. A permanent compliance loss from $c_0 \to c_1 < c_0$ shifts the entire curve downward one-for-one. Holding demography fixed at $d = d^*$ the replacement rate drops multiplicatively according to equation (9) therefore moving from the pre-shock optimum characterized by point A to the post shock optimum point B. Economically, c is the effective coverage of the contribution base thus when fewer contributors comply, the same contribution pay roll s must finance the same amount of retirees therefore generosity falls in exact proportion unless the contribution rate or demography is adjusted via policy measures.

Re-expressing the same identity as $\rho(x) = (sc)x$ with $x = 1/d$ turns the budget hyperbola into a ray through the origin whose slope equals the effective contribution base sc. A compliance shock rotates the ray downwards with a fall in slope from $sc_0 \;\to\; sc_1$. Evaluated at the same demographic point $x^* = 1/d^*$ the vertical impact $\rho_1 = \rho_0(c_1/c_0)$ coincides with Panel A. All policy levers that raise sc like a higher contribution rate, better social security enforcement or widening the contributor base produce a rotation in the slope. Demographic reform that lowers d shift the evaluation point rightward along the ray.

3.2. Steady state policy to keep the target replacement rate

For tractability, we posit a single permanent exogenous compliance shock at date τ:

$$c_t = \begin{cases} c_0, & t < \tau \\ c_1 < c_0, & t \geq \tau \end{cases}.$$

Thus, after the shock the compliance rate is fixed at c_1 and does not respond to policy choices f' or s'. This isolates the pure mechanics of pensionable-age and contribution-rate adjustments, extensions that endogenize c_t are left for future work.

Fix a target replacement $\bar{\rho} > 0$, e.g. the pre-shock value $\rho^* = \bar{\rho}$. After the shock the target rate can be stabilized again through policy interventions on the contribution share and the retirement age. All policy pair (s', f') that meet $\rho^* = \bar{\rho}$ satisfying the policy frontier

$$s(f') = \alpha \frac{f'}{n + m - f'}, \quad 0 < f' < n + m. \qquad (10)$$

Equation (10) with $\alpha = \bar{\rho}/c_1$ is the complete steady-state feasibility locus for a given target replacement rate of $\bar{\rho}$: every admissible (s', f') pair that sustains the target must lie on this curve.

In (s', f')-space the frontier is strictly increasing and convex in $f' \in (0, n+m)$ with slope

$$\frac{ds'}{df'} = \alpha \frac{n+m}{(n+m-f')^2} > 0,$$

and curvature

$$\frac{d^2s'}{df'^2} = 2\alpha \frac{n+m}{(n+m-f')^3} > 0.$$

It passes through the origin as $f' \downarrow 0$ and $s' \downarrow 0$ (non-existence of a PAYG system) and has a vertical asymptote at $f' = n + m$. Optimal mixes are tangencies between this frontier and sets of policy cost functions as pictured in figure 6 based on our calibration; if tangency is infeasible, the Karush-Kuhn-Tucker (KKT) corners occur exactly where the frontier meets a policy bound like s_{max} or f_{min}. A higher retirement inflow f' means more beneficiaries today and fewer contributors tomorrow, because those who retire leave the contributor base. To keep the same target replacement rate $\bar{\rho}$, the contribution rate must rise. In real economies, lowering the pensionable age therefore always pushes up the payroll wedge required to finance the same generosity - there is no offsetting channel in PAYG accounting. Convexity says the marginal contribution increase needed for an extra bit of early retirement accelerates as f' gets larger (or equivalently, as the system gets "older"). Raising f' does two things at once - it adds retirees (numerator effect) and removes workers from the contribution base (denominator effect). This double-hit makes financing costs rise more than proportionally as you keep pushing f' up. Empirically, systems that allow very early or generous exit see contribution needs climb steeply, constant with the convex shape. Its monotonicity reflects the unavoidable PAYG trade-off (more retirees per contributor → more payroll needed). Its convexity reflects the compounding fiscal strain of earlier exit: simultaneously expanding claims and shrinking the base. This matches observed pressures in ageing systems: once retirement inflows are high, holding generosity constant requires sharply rising payroll wedges or, conversely, large age reforms to avoid explosive contribution needs.

The elasticity of s to f on the frontier is

$$\frac{d \ln s'}{d \ln f'} = \frac{n+m}{n+m-f'} > 1.$$

When f' is low (old-age entry is tight), a 1% further tightening buys more than a 1% reduction in s': diminishing returns to ever-later retirement ages. When f' is high, a 1% relaxation (higher f') demands more than a 1% increase in s': costs snowball. As $f' \to n + m$ (the viability boundary), $\varepsilon_{s,f} \to \infty$ and $s' \to \infty$: the system becomes infeasible at fixed $\bar{\rho}$.

3.2.1. s'-only policy after a permanent c-shock

Consider the system at $t = \tau$ when c falls from c_0 to c_1. With the budget identity $\rho_t d_t = s_t c_t$, the impact of the shock is characterized through equation (9). An authority that insists on protecting generosity on impact, no cut in the replacement rate ρ, can do so with contributions by imposing the replacement targeting rule

$$s_t = \frac{\bar{\rho} d_t}{c_1} \text{ for all } t \geq \tau \quad (11)$$

with $\bar{\rho}$ the announced target e.g. the pre-shock level ρ_t. This rule is necessary and sufficient to keep $\rho_t \equiv \bar{\rho}$ every period. With f unchanged and the economy at the pre-shock steady state, $d_t \equiv d^*(f_0)$ so $\bar{\rho} \equiv s_0 c_1/d^*$ thereafter.

Under the feedback rule (11), the optimal contribution rate is defined as

$$s_t = \frac{\bar{\rho} d_t}{c_1} \to \frac{\bar{\rho} d_0^*}{c_1} = s_0 \frac{c_0}{c_1} = s', \quad (12)$$

the long-run contribution rate policy s'needed to sustain the target replacement rate at the steady state.

Looking at the feasible policy screen, keeping ρ_t flat via (11) is feasible if the statutory cap satisfies

$$s_{max} \geq \frac{\bar{\rho}}{c_1} \sup_{t \geq \tau} d_t = \frac{\bar{\rho}}{c_1} \max\{d_\tau, d_0^*\}.$$

To keep benefits stabilized at $\bar{\rho}$ after the compliance drop to c_1, the contribution rate must scale one-for-one with the dependency ratio $s_t = (\bar{\rho}/c_1) d_t$; feasibility therefore requires that the statutory cap s_{max} is high enough to cover the demographic burden encountered at the shock. Either the impact burden d_τ or the long-run level d_0^*, whichever is larger. If this fails, a s-only impact stabilization does not work, simultaneous tightening of the pensionable-age f is unavoidable to reduce the dependency ratio so that the required statutory rate remains within the feasible policy screen.

The s'-only strategy places all short run adjustment on workers and firms via a higher payroll wedge, fully protecting current retirees with no cut in pension benefits. Its drawbacks are the rise in the labor wedge and possible caps that bind precisely when the compliance shock is huge. Efficiency and political costs of an s' policy will be addressed when looking at a (s', f') policy mix in chapter 3.3.

3.2.2. f'-only policy after a permanent c-shock

Suppose policymakers refuse to raise the contribution rate ($s_t \equiv s_0$) and instead tighten the pensionable age with a f'-only policy. Because d_τ is predetermined, (9) implies an unavoidable drop in generosity:

$$\rho_{\tau^+} = \frac{s c_1}{d_\tau} < \rho_{\tau^-}.$$

What can be achieved is a full restoration in the long run by choosing a new constant retirement inflow rate f' that hits the target $\bar{\rho}$ in steady state:

$$\bar{\rho} = \frac{s_0 c_1}{d^*(f')}, \qquad d^*(f') = \frac{f'}{n + m - f'} \to f' = \frac{(n+m)s_0}{\alpha + s_0}, \alpha \equiv \frac{\bar{\rho}}{c_1} \quad (13)$$

This f' is lower than f_0 when $c_1 < c_0$ (tighter pensionable age); feasibility requires $f_{min} \leq f' < \min\{f_0, n+m\}$.

Dynamics with $f_t \equiv f'$ show demography becomes $d_{t+1} = a' d_t + b'$ with $a' = \frac{1-m}{1+n-f'} \in (0,1)$.

Hence, under the constant post-reform retirement inflow rate $f_t \equiv f'$, the dependency ratio converges to its new steady state $d_t \to d^*(f')$, and the replacement rate correspondingly rises according to $\bar{\rho} = (s_0 c_1/d_t)$, increasing monotonically toward the target $\bar{\rho}$ at the geometric rate a'.

Notably, tightening pensionable age both lowers the target dependency level $d^*(f')$ and accelerates convergence by reducing a'.

Even an extreme, feasible reduction in f_τ cannot repair ρ on impact because the state is predetermined:

$$d_{\tau+1} \geq \frac{(1-m)d_\tau}{1+n} \;\rightarrow\; \rho_{\tau+1} \leq \frac{s_0 c_1}{\frac{(1-m)d_\tau}{(1+n)}}.$$

This inequality formalizes that demographic adjustment is intrinsically sluggish relative to within-period fiscal instruments: because the dependency ratio is a state variable that cannot jump, an f'-only response cannot offset the revenue loss on impact, making an immediate reduction in benefits unavoidable.

The f'-only strategy places the short-run burden on current and near-retirement cohorts (delayed eligibility), with no rise in the payroll wedge for workers. It is the system's structural sustainability lever: over time it shrinks the retiree-to-contributor burden, lifting ρ back to the target without raising s. Its drawbacks are political and legal limits on pensionable- age reform and the absence of impact stabilization.

To summarize an s'-only policy is the impact instrument. It can exactly neutralize a compliance shock today via (11), converging to the steady state (12). It is constrained by caps and the political cost of a higher labor wedge. On the other hand an f'-only policy is the sustainability instrument. It cannot protect and repair the impact in the short run, but it fully restores the target in the long run via (13) and does so while accelerating the speed of demographic repair.

These two isolated responses make the two mechanisms of the PAYG system transparent – budget versus demography – and set up why a policy mix of contribution rate and retirement age adjustment (s', f') strictly dominates.

3.3. Optimal policy mix after a permanent compliance shock c

We analyze the optimal joint adjustment of the contribution rate and pensionable-age policy (s', f'), in response to a permanent compliance deterioration $c_0 \downarrow c_1 (c_0 > c_1)$. The government commits to a target replacement rate $\bar{\rho} > 0$ e.g. the pre-shock steady state. We keep demographic primitives (n, m) constant and maintain the viability conditions $0 < f < n + m$ and $1 + n - f > 0$.

All constant pairs (s', f') that sustain the target $\bar{\rho}$ in steady state lie on the policy frontier (10). Let C_s and C_f be strictly convex and continuous, capturing economic and political costs of moving s and f away from status quo (s_0, f_0):

$$C_s = \frac{\phi_s}{2}(s - s_0)^2 \text{ and } C_f = \frac{\phi_f}{2}(f - f_0)^2 \quad (14)$$

Equation (14) is intended as a local second-order approximation to adjustment and implementation costs around the status quo capturing political, administrative, and transitional frictions of changing s and f so it penalizes deviations in either direction without implying that lower contribution rates or earlier retirement are intrinsically undesirable.

With statutory bounds $s \in [s_{min}, s_{max}], f \in [f_{min}, f_{max}] \subset (0, n + m)$, the steady state policy problem

$$\min_{s', f'} C_s(s') + C_f(f') \quad s.t. \quad s' = \alpha \frac{f'}{n + m - f'} \quad (15)$$

has a unique solution f^*. The bounds $s \in [s_{min}, s_{max}]$ and $f \in [f_{min}, f_{max}]$ are imposed purely as implementability constraints reflecting political limits and are not strictly needed to get uniqueness. Whenever the unconstrained tangency lies in the interior, the solution coincides with the unique interior minimizer, and only if a constraint bind does the optimum move to the corresponding boundary (a corner solution in the standard KKT sense).

If it is interior $f_{min} < f^* < f_{max}$ the optimum satisfies the tangency slope-matching condition

$$\alpha \frac{n+m}{(n+m-f^*)^2} = -\frac{\phi_f}{\phi_s}\frac{f^*-f_0}{s(f^*)-s_0} \quad (16)$$

equivalently

$$\phi_f(f^*-f_0) + \phi_s(s(f^*)-s_0)s(f^*) = 0 \text{ with } s(f^*) = \alpha\frac{n+m}{(n+m-f)^2} > 0.$$

If bounds bind, f^* is the minimizer of the same strictly convex objective on the restricted interval (first-order condition replaced by Karush-Kuhn-Tucker, see Appendix). After a compliance decrease $\alpha \uparrow$, one has $s(f_0) > s_0$. At any interior optimum this implies $f^* < f_0$ and $s(f^*) > s_0$. The planner shares the burden between a higher pensionable age and a higher long-run contribution rate rather than choosing a corner.

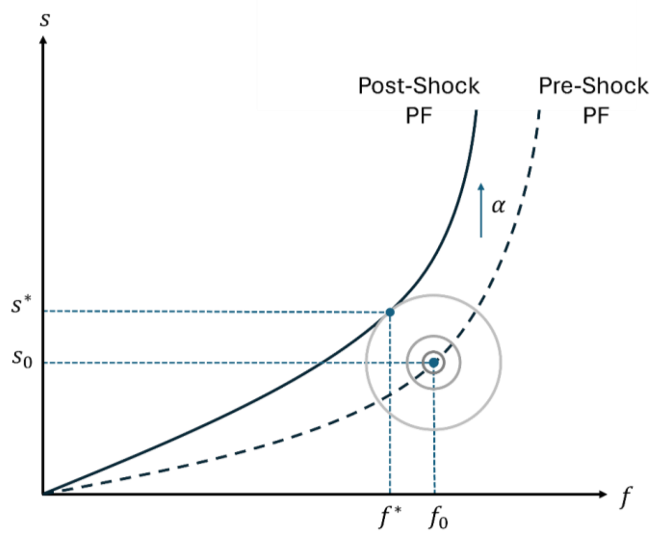

Figure 2. Visualization of the Slope Matching Condition Equation (16)

Theorem 1:

(i) The feasibility set is nonempty if the frontier intersects the admissible policy sets defined by the statutory bounds; in that case, problem (15) admits a unique solution.

(ii) If interior, it is characterized by the tangency condition

$$C_f'(f') + C_s'(s')\alpha\frac{n+m}{(n+m-f')^2} = 0, \quad (17)$$

with $s' = \alpha\frac{f'}{(n+m-f')}$.

(iii) If the interior point violates bounds, the KKT solution is the boundary point on the frontier with the lowest $C_s + C_f$; equivalent, minimizing the one dimensional convex $F(f)$ $:= C_f(f') + C_s\left(\frac{\alpha f}{n+m-f}\right)$ on the induced interval

$$f \in [L, U], L = \max\left\{f_{min}, \frac{(n+m)s_{min}}{\alpha + s_{min}}\right\}, U = \min\left\{f_{max}, \frac{(n+m)s_{max}}{\alpha + s_{max}}\right\}.$$

The interior condition equates marginal policy-cost ratio to the technological slope of the feasibility frontier. Because the frontier is strictly increasing and convex, a unique tangency exists unless bounds bind; hence a genuine mix is generically optimal. Theorem 1 characterizes the long-run policy mix by minimizing the steady-state component of the policy-loss function subject to the steady-state PAYG frontier, not the costs during transition to the steady state. This abstraction deliberately omits transitional welfare and adjustment costs along the convergence path.

This subsection studies a tractable rule-based benchmark to characterize transitional mechanics and convergence. The fully optimal intertemporal adjustment under convex policy costs is derived later in Theorem 2. After a permanent compliance loss to c_1, consider the feedback rule from equation (11) and a constant retirement-entry intensity $f_t = f' \in (0, n + m)$. Then the period budget identity implies on-target stabilization each period to secure the target replacement rate. Demography follows the one-state law equation (5). Hence d_t converges monotonically with no overshooting to

$$d^*(f') = \frac{f'}{n + m - f'},$$

And the payroll sequence inherits the same geometric speed,

$$s_t = \frac{\bar{\rho} d_t}{c_1} \downarrow s_\infty \text{ with } s_\infty = \frac{\bar{\rho} d^*(f')}{c_1}.$$

The half-life of the gap in either d_t or s_t is

$$t_{1/2} = \frac{\ln 2}{\ln\left(\frac{1}{D_d}\right)} = \frac{\ln 2}{\ln(1 + n + m - f')}.$$

The choice of f' determines the long run demography. The feedback mechanism (11) pins ρ_t to a target every period and lets the payroll rate decline endogenously as the dependency ratio improves. One-dimensional linear adjustment with slope $D_d \in (0,1)$ delivers monotone convergence and a close form half-life.

Given any accuracy target $\varepsilon > 0$ for the dependency ratio's state to be "effectively" at its steady state, the tolerance bound is the smallest horizon T_ε such that the trajectory is ε-close to steady state. Thus, years to ε-near stabilization after the c-shock and policy intervention is given as

$$T_\varepsilon = min\{k \in \mathbb{N} : |d_{\tau+k} - d^*| \leq \varepsilon\} = \left\lceil \frac{\ln\left(\frac{\varepsilon}{|d_\tau - d^*|}\right)}{\ln \lambda} \right\rceil.$$

Under the feedback rule $s_t = \bar{\rho} d_t / c_1$, the same bound applies to $|s_t - s_\infty|$.

In the quadratic benchmark and conditional on an interior post-shock solution with $s' > s_0$, we obtain

$$\frac{\partial s'}{\partial \alpha} > 0, \frac{\partial f'}{\partial \alpha} < 0.$$

A larger shock, higher α, steepens the policy frontier, holding costs fixed, the optimum shifts toward contribution policy s', using less tightening of the pensionable age f'. If caps on s are tight, the optimum hits the corner $s' = s_{max}$ and requires a minimum $f' \geq \frac{(n+m)s_{max}}{\alpha + s_{max}}$.

If the government insists on no impact cut in the replacement rate, the within-period targeting rule (11) holds for all t. Furthermore, the demography remains on the law of motion (5). The dynamic policy problem is then:

$$\min_{\{f_t\}} \sum_{t=\tau}^{\infty} \beta^{t-\tau} \left[\underbrace{\frac{\phi_s}{2}\left(\frac{\bar{\rho}}{c_1} d_t - s_0\right)^2}_{\substack{cost\ of\ contribution \\ deviation}} + \underbrace{\frac{\phi_f}{2}(f_t - f_0)^2}_{\substack{cost\ of\ retirement \\ age\ deviation}} \right] s.t.\ (11). \quad (18)$$

Theorem 2:

Given $f \in [f_{min}, f_{max}] \subset (0, n+m)$, $\beta \in (0,1)$ and $\phi_f, \phi_s > 0$, (18) has a unique optimal stationary Markov policy[1]. The necessary and sufficient first conditions with multipliers μ are

$$\phi_f(f_t - f_0) + \mu_{t+1}\frac{1+n+(1-m)d_t}{(1+n-f_t)^2}$$

$$\mu_t = \beta\left[\phi_s\frac{\bar{\rho}}{c_1}\left(\frac{\bar{\rho}}{c_1}d_t - s_0\right) + \mu_{t+1}\frac{1-m}{1+n-f_t}\right]$$

with standard transversality condition. The conditions characterize the interior solution; if the bounds $f \in [f_{min}, f_{max}]$ respectively $s \in [s_{min}, s_{max}]$ bind, the corresponding KKT multipliers enter the first-order conditions, and the usual complementary-slackness conditions apply.

Because the post-shock environment is stationary and Theorem 2 delivers an optimal stationary Markov policy, the induced path (s_t, f_t, d_t, μ_t) converges to a time-invariant point. This limiting allocation coincides with the steady-state minimizer characterized in Theorem 1 and the stationary Euler conditions determine the associated shadow value μ^* consistent with the transversality condition.

Rule (11) uses the contribution rate to stabilize the replacement rate ρ_t on impact, while the Euler system governs the intertemporal allocation of retirement entry adjustments. It tilts the path of f_t to gradually compress the dependency ratio d_t and thereby relax the contribution requirement in later periods. When contribution adjustments are relatively costly, high ϕ_s/ϕ_f, the optimal transition front-loads retirement tightening leading to a stronger early reduction in f_t. Conversely, when pensionable age adjustments are relatively costly, high ϕ_f/ϕ_s, the transition relies more heavily on s_t and smooths changes in f_t.

Linearize the frontier at (f_0, s_0, α_0): $\Delta s = g_\alpha \Delta\alpha + g_f \Delta f$ with

$$g_f = \alpha_0\frac{n+m}{(n+m-f_0)^2};\ g_\alpha = \frac{f_0}{n+m-f_0};\ \Delta\alpha > 0.$$

With symmetric quadratic costs, the optimal split is

$$\Delta s^* = \frac{g_\alpha}{1+g_f^2}\Delta\alpha;\ \Delta f^* = -\frac{g_f g_\alpha}{1+g_f^2}\Delta\alpha. \quad (19)$$

The total cost falls by the factor $1/(1+g_f^2)$ relative to s-only policy. The frontier slope g_f, representing steepness of substituting f for s, is the only statistic governing the sharing rule locally: when the system is "old" (frontier steep), optimal policy uses more pension-age reform and less contribution-rate adjustment.

For the contribution cap under dynamic targeting, feasibility requires

$$s_t = \frac{\bar{\rho}}{c_1}d_t \le s_{max}\ \forall t\ \rightarrow \bar{\rho} \le \frac{s_{max}c_1}{\sup_t d_t}.$$

If violated, the mix must include immediate f-tightening to reduce $\sup_t d_t$. For age bounds, steady-state feasibility with $s \le s_{max}$ requires

$$f' \ge \frac{(n+m)s_{max}}{\alpha + s_{max}}.$$

If biological or legal limits imply $f_{min} > (n+m)s_{max}/(\alpha + s_{max})$, the target $\bar{\rho}$ is infeasible at given c_1.

[1] A Markov (state-feedback) policy maps the current state d_t into the control f_t. With time-invariant primitives and $\beta < 1$, the Bellman operator is a contraction, so an optimal stationary Markov policy exists. Strict convexity in f makes the policy unique.

With strictly convex policy costs and feasible bounds, the optimal solution to (16) is generically a strict interior mix, relative to either s-only or f-only policy. It achieves the target replacement rate $\bar{\rho}$ at strictly lower total policy costs. Under replacement targeting, the optimal dynamic mix (18) keeps ρ_t flat now while minimizing the discounted costs of moving d_t so that the required $s_t = \bar{\rho} d_t / c_1$ declines over time.

The mix aligns instruments with their clocks: use s for impact stabilization and f for demographic repair. The convex shape of the frontier ensures that sharing the burden across the two levers is cheaper than leaning on a single lever, except when hard bounds force corners.

A permanent compliance shock shifts the budget set down. The policy frontier translates the target replacement rate into feasible (s', f') policy pairs. Convex policy costs and the frontiers geometry deliver a unique least-cost-mix in steady state, and a unique optimal path under dynamic targeting. The analytics yield implementable rules: (i) an automatic stabilizer to protect generosity on impact and (ii) a calibrated path for pensionable-age that brings the dependency ratio down, unwinding the required contribution rate. The mix is thus the neutral, cost-efficient response to a compliance shock, robust to bounds as well as transparent to communicate and fully grounded in the PAYG structure.

4. Quantitative illustration and policy experiments

This section feeds the model with numbers. We first pin down a baseline steady state that matches pre-shock institutional targets and demographics contribution rate s_0, compliance rate c_0 and target replacement rate $\bar{\rho}$. The overall aim of the government is to design policy measures to keep the replacement rate stable for the purpose of intergenerational fairness or political economic reasons. Thus, feasible policy instruments are the contribution rate s or the suspension of the pensionable age mapped via the retirement inflow rate f. Therefore, a decrease in the retirement inflow rate is equivalent to an increase in the pensionable age e.g. from age 65 to age 67. Demography is summarized by the dependency ratio equation (6). The baseline satisfies the accounting identity $\bar{\rho} d^* = s_0 c_0$ and anchors the dynamic law of motion for d_t.

We then study a permanent compliance shock $c_0 \rightarrow c_1 < c_0$ and four policy regimes: (i) no intervention, (ii) s-only intervention, (iii) f-only intervention and (iv) a policy mix (s', f') chosen on the policy frontier equation (10) with equal policy costs scenario A. Additionally, two scenarios for relative costs differentials are analyzed: (B) f adjustment costly $\phi_f \gg \phi_s$ and (C) s adjustment costly relative to f ,$\phi_s \gg \phi_f$.

Table 1 lists primitives, policy baselines and calibration-specific weights. All subsequent figures use these values to calibrate the baseline, implement the shock, and compute the least cost policy mix via the tangency between the iso-cost sets and the post shock frontier.

Table 1. Baseline calibration parameters

Variable	Value	Description
T	60	Time horizon (in years)
t_{shock}	10	Time shock occurs
n	0.01	Contributor growth p.a.
m	0.04	Retiree exit rate p.a.
d^*	0.5	Baseline dependency ratio
f_0	0.0167	Retirement inflow rate p.a.
s_0	0.2	Baseline contribution rate
c_0	0.9	Baseline compliance rate
c_1	0.75	Shock compliance rate
ρ^*	0.36	Target replacement rate
α	0.48	Policy frontier parameter $\rho/c_{0,1}$
$\phi_s^A = \phi_f^A$	1	Policy costs of contribution rate adjustment – scenario A
ϕ_s^B	1	Policy costs of contribution rate adjustment – scenario B
ϕ_s^C	100	Policy costs of contribution rate adjustment – scenario C
ϕ_f^B	100	Policy costs of extending the pensionable age – scenario B
ϕ_f^C	1	Policy costs of extending the pensionable age – scenario C

A 60-year horizon is long enough to span the medium-run adjustment of a mature PAYG system and to capture the demographic swing that dominates pension finance in long-horizon projections. T_t also corresponds to roughly two human biological generations, using the demographic convention of an intergenerational interval of about ~30 years proposed by Tremblay and Vézina (2000). A retirement exit rate of $m = 0.04$ implies an average expected retirement duration of 25 years accounting for projected longevity (Rotschedl et al. 2024). Consistent with the demographic baseline in the latest European Commission 2024 Ageing Report, we set $d^* = 0.5$ to anchor the model to current PAYG pressure in a mature European system: for Germany, the report's country fiche reports a pension-system dependency ratio of 0.5 in 2022. This choice is therefore not a convenient normalization but an empirically grounded starting point. We treat n as effective growth of the contribution-paying base. This is intentionally moderate: the Ageing Report (2024) states rising participation/employment rates but also a shrinking working-age population, so a small positive n is a neutral middle ground that avoids hard-wiring pessimism into the baseline and keeps the post-shock dynamics driven by the compliance disturbance rather than by a collapsing tax base. The replacement rate $\rho = 0.36$ is set accordingly to the published replacement rate of the German public pension (earnings-related) in 2022 with 36.8% while the statutory contribution rate of $s = 0.2$ approaches the aggregate contribution of 18.6% in Germany while EU-wide the aggregate is higher, which makes our determination appear to be moderately average. Interpreting compliance as the fraction of the statutory contribution base that is actually remitted according to Bailey and Turner (1997), $c_0 = 0.9$ corresponds to a leakage of about 10%, squarely in the range suggested by Germany's shadow-economy magnitudes often around 10% of GDP in recent estimates by Schneider and Boockmann (2023), which are a natural empirical proxy for under-reported wage bases and undeclared work. All other parameters are either endogenous given by the model or exogenously set.

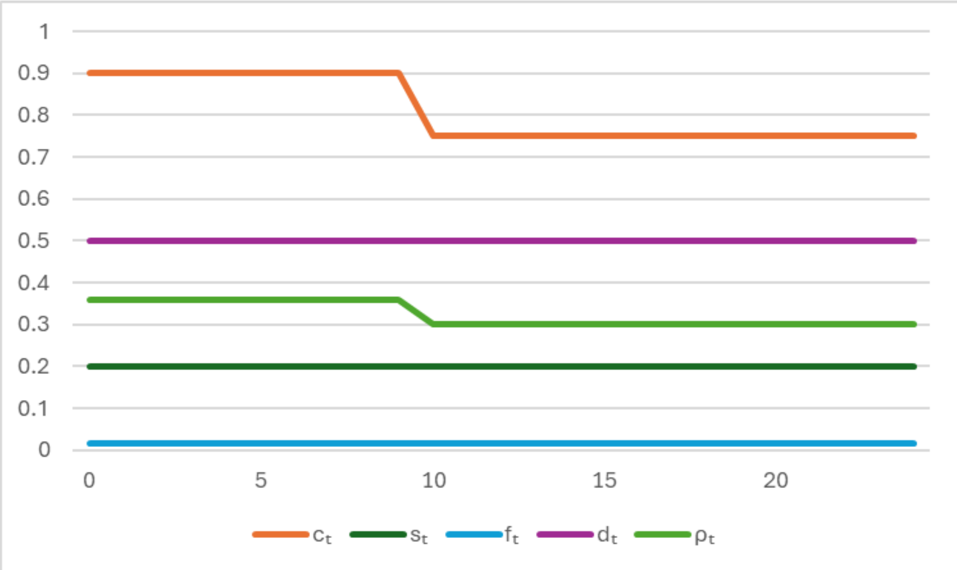

Figure 3. Permanent compliance shock with no intervention

Figure 3 plots the no-policy response to a permanent fall in the compliance rate at $t = 10$. With instruments held fixed $s_0 = s_t$ and $f_0 = f_t$, the period PAYG identity $\rho_t d_t = s_t c_t$ implies a pure impact effect. The replacement rate drops discretely from $\rho_0 = s_0 c_0 / d^*$ to $\rho_1 = \rho_0(c_1/c_0)$ and remains at lower plateau thereafter. In our calibration $c_0 = 0.9 \rightarrow c_1 = 0.75$ generosity falls from 0.36 to 0.3 (-16.67%), exactly mirroring the compliance loss, an immediate corollary of the unit elasticities $\varepsilon_{p,c} = 1$ and $\varepsilon_{p,d} = -1$ at fixed s and d. Economically, the shock reduces the effective contribution base while the retiree burden is unchanged. Absent policy intervention, the system instantaneously reprices annuities downward, with no transitional dynamics because f is fixed. This benchmark pins down the magnitude of the policy problem and provides a sufficient statistic for counterfactual design. Stabilizing $\rho = \bar{\rho}$ after c_1 requires either an s-only move $s' = s_0(c_0/c_1)$, a demografic adjustment $d' = s_0 c_1/\bar{\rho}$ via f, or a mix (s', f') on the policy frontier.

First, we study the counterfactual in which the policymaker keeps generosity at its target $\bar{\rho}$ by adjusting only the contribution rate s, holding demography and therefore pension entry fixed. The period budget identity together with the target replacement rate $\rho = \bar{\rho}$ implies a required payroll adjustment

$$s' = \frac{\bar{\rho} d^*}{c_1} = s_0 \frac{c_0}{c_1}.$$

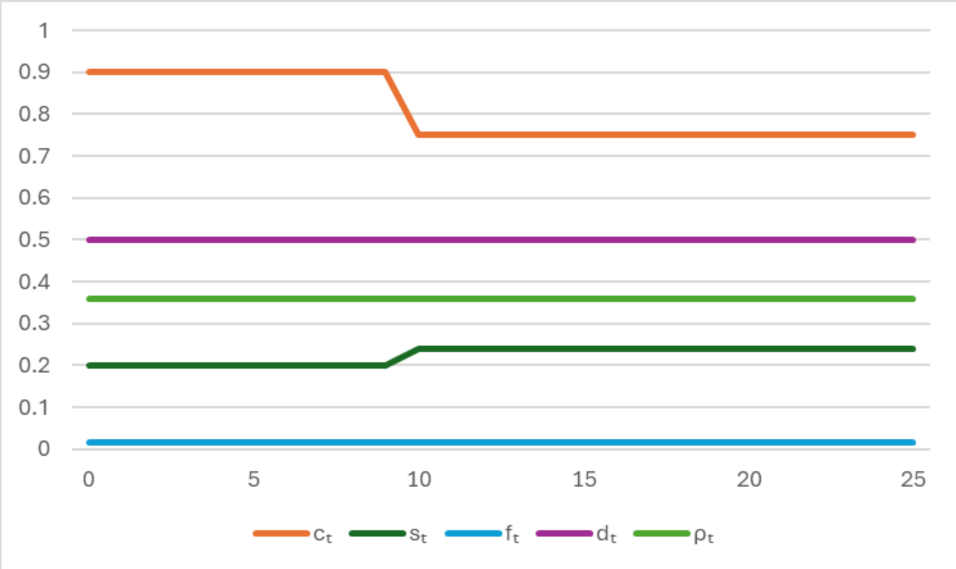

Figure 4. Permanent compliance shock s′-only intervention

Hence the elasticity of the required contribution rate with respect to compliance requires a 1% increase in s for a 1% decrease in c. Under our calibration this yields to a permanent and immediate contribution rate policy of $s' = 0.24$ (+20%) to offset the negative impacts on generosity. Figure 4 displays these dynamics with a permanent compliance shock at $t = 10$. The policy rule implements the steep increase in $s_t: s_0 \rightarrow s'$ at the shock date, while the retirement-entry rate f_t and the dependency ratio d_t remain unchanged. Because the identity binds each period, generosity is fully stabilized on impact $\rho_t = \bar{\rho}$ for all t. There is no transition in ρ_t and the new contribution wedge is permanent. The simulation isolates the fiscal arithmetic of enforced losses: when compliance erodes, stabilizing benefits through s alone requires a proportional, immediate rise in the payroll rate. Conversely, any hesitation or partial adjustment translates one-for-one into a shortfall of generosity, or the need of replacement rate financed by governmental expenditures. From a policy perspective, the s-only response is quickly implementable and transparent but potentially costly since it increases the labour tax wedge and may depress labour supply or future compliance which are both held exogenously constant in this model.

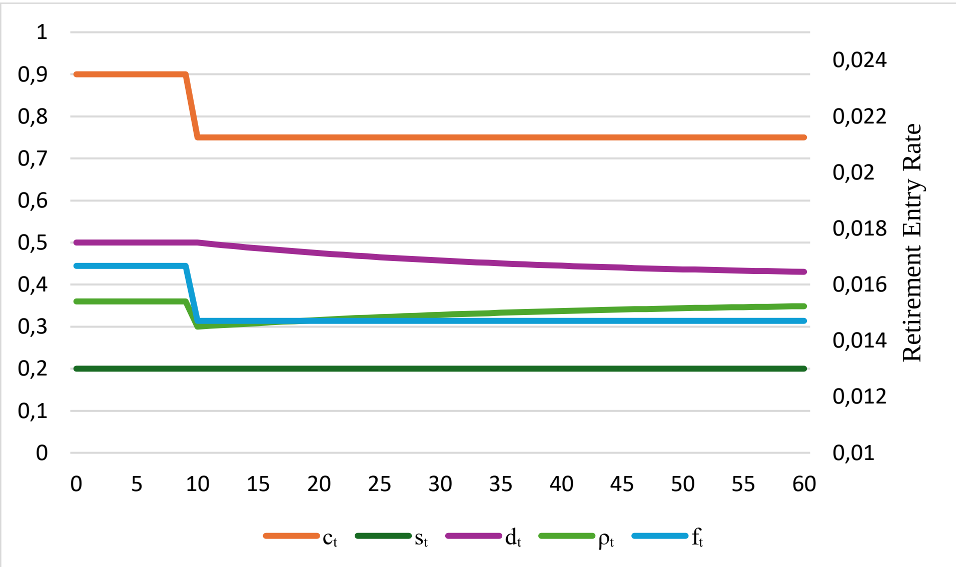

Figure 5. Permanent compliance shock f′-only intervention

In *Figure 5* the government keeps the contribution rate fixed at its pre-shock level $s_t = s_0$ and restores generosity entirely by tightening the retirement entry f. Because demography is a state, the dependency ratio d_t cannot jump hence the period by period budget identity $\rho_t d_t = s_0 c_t$ implies an impact drop $\rho_{10} = \rho_0 (c_1/c_0)$. As a response to the compliance shock, the authority sets a lower, constant retirement-inflow rate $f' < f_0$, which is equivalent to a higher retirement age, from $t = 10$ onwards. The induced demographic law of motion equation (5) monotonically reduces the dependency ratio $d_{t+1} = D(d_t, f')$ so that $\rho_t = s_0 c_1 / d_t$ drifts back up toward the target replacement rate. The steady-state requirements pin down thefinaldemography d^* and the implied entry flow:

$$d^* = \frac{s_0 c_1}{\bar{\rho}};\ d^* = \frac{f'}{n + m - f'} \ \rightarrow\ f' = \frac{(n+m)d^*}{1 + d^*}$$

Thus the whole adjustment burden is borne by lowering d rather than raising s. *Figure 4* shows exactly this pattern: (i) c_t drops at $t = 10$; (ii) s_t remains flat at s_0; (iii) f_t falls downward at the shock; (iv) the dependency ratio d_t declines gradually toward d^*; and (v) the replacement rate rises smoothly from its impact level back to its target value. There is no overshooting because $d_{t+1} = D(d_t, f')$ is a contraction under our calibration; the speed of convergence and half-life are governed by the slope $D_d \in (0,1)$, which increases if the retirement-age change is modest and decreases if f' is tightened more aggressively. A pure retirement-age response stabilizes generosity without raising the payroll wedge, but unlike the s-only rule it achieves stabilization gradually through the demographic state leading to a generosity deficit of $|\rho^* - \rho_t|$ each period. The trade-off is transparent: larger cuts in f yield faster return to ρ_t to its target but at higher adjustment costs, milder changes prolong the shortfall. This motivates the least-cost policy mix on the post-shock frontier, studied next.

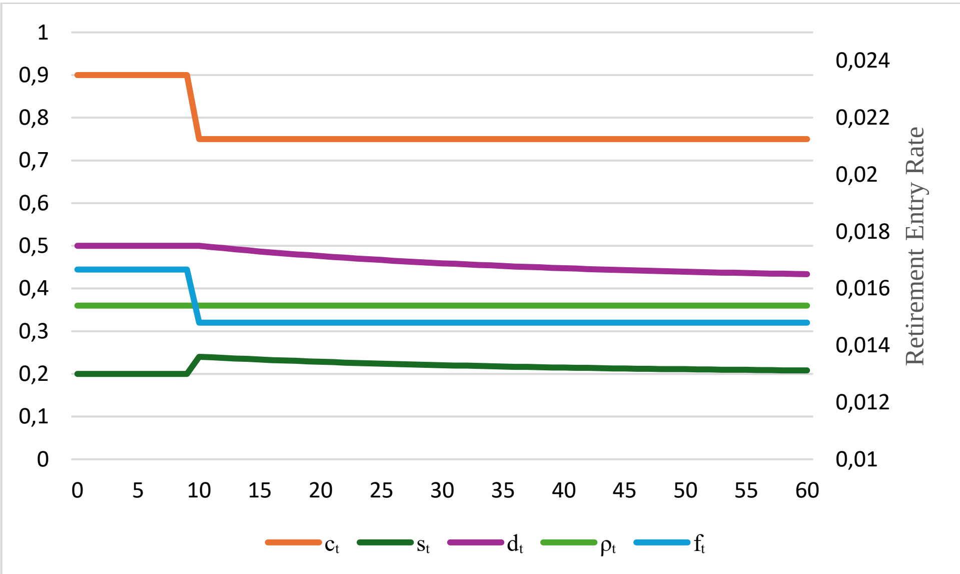

Figure 6. Permanent compliance shock s', f'-mixed intervention with equal costs

We now consider the policy that uses both instruments s and f to absorb a permanent compliance loss $c_0 \rightarrow c_1 < c_0$ when adjustment costs ϕ_s and ϕ_f are symmetric which we refer to as scenario A. At the shock date $t = 10$ the demographic state cannot jump, so the period PAYG budget identity $\rho_t d_t = s_t c_t$ implies that the unique impact policy that prevents any generosity shortfall given equation (12). From the following period onward the government permanently tightens retirement entry to $f' < f_0$ resulting in a higher pensionable age. Given constant f', the dependency ratio evolves according to the one-state law of motion

$$d_{t+1} = D(d_t, f') = \frac{(1-m)d_t + f'}{1+n-f'}$$

which is a global contraction because $D_d = \frac{1}{(1+n+m-f')} \in (0,1)$. Along side the transition we impose the simple feedback rule

$$s_t = \frac{\bar{\rho} d_t}{c_1} \; for \; t \geq 10$$

so that $\rho_t \equiv \bar{\rho}$ in every period. The mixed policy combines impact stabilization of benefits with a gradual erosion of the payroll rate as demography improves in favor of the budget. Convergence is monotonie and its speed is governed by the contraction factor D_d. The half-life of the gap in either d_t or s_t equals $t_{1/2} = \ln 2 \; / \; \ln(1/D_d)$ and shortens with a more ambitious reduction in f'.

In the long run the feasible stationary pairs satisfy the post-shock policy frontier

$$s(f') = \alpha \frac{f'}{n+m-f'} \quad with \;\; \alpha \equiv \frac{\bar{\rho}}{c_1},$$

which is strictly increasing and convex on $f' \in (0, n+m)$. With equal policy costs $\phi_s = \phi_f = 1$,

$$C(f', s') = \frac{1}{2}(f' - f_0)^2 + \frac{1}{2}(s' - f_0)^2,$$

the optimal stationary policy mix (f', s') is the unique interior tangency between the iso-cost ellipse centered at (f_0, s_0) and the frontier. The slope-matching condition,

$$\frac{ds'}{df'} = \alpha \frac{n+m}{(n+m-f')^2} = -\frac{\frac{\partial C}{\partial f'}}{\frac{\partial C}{\partial s'}} = -\frac{f' - f_0}{s' - s_0},$$

immediately yields the economically intuitive signs $(f' - f_0) < 0$ and $(s' - s_0) > 0$. The burden is shared between a higher retirement age and a smaller permanent rise in the payroll rate than under an s-only response. Because both the frontier and the iso-cost sets are strictly convex, this tangency strictly dominates corner solutions. It minimizes policy effort for a given target $\bar{\rho}$ and therefore yields $C_{mix} < \{C_{s-only}, C_{f-only}\}$.

For policymakers the mixed rule is compelling on both positive and normative grounds. It guarantees no temporary or permanent fall in generosity thus current retirees are protected, yet it reduces the long-run tax wedge relative to s-only by off-loading part of the adjustment to the demographic state. At the same time, it avoids the transitional shortfall in the replacement rate that is inherent to f-only policy. Implementation is straightforward, an immediate, transparent adjustment of s at the shock date and a legislated, permanent shift in the pensionable age while the feedback is $s_t = \bar{\rho} d_t / c_1$. Under symmetric costs this interior solution is therefore the cost-minimizing and welfare-dominant way to absorb a compliance shock. Alternative cost weights simply tilt the iso-costs and move the tangency along the post-shock frontier as we can see in the following.

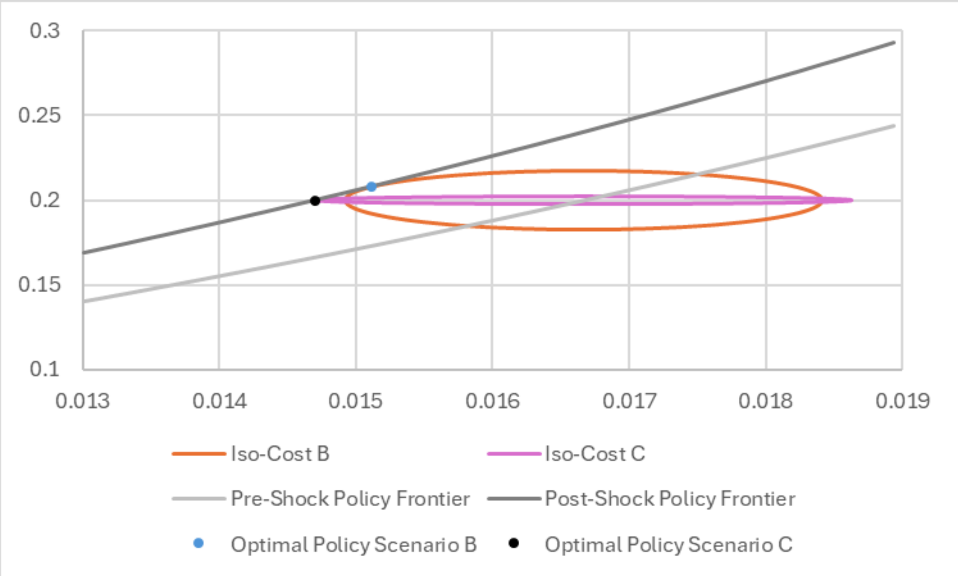

Figure 7. Permanent compliance shock (s', f')-mixed intervention with different policy cost weights

Figure 6 combines the pre-shock and post-shock policy frontier in (s, f)-space with two iso-cost sets that encode alternative relative adjustment costs ϕ_s and ϕ_f. The frontier includes all instrument pairs that sustain the target replacement rate $\bar{\rho}$ at compliance rate c according to $s'(f')$. A compliance loss shifts the frontier upward and makes it uniformly steeper since α scales both the level and slope of $s(f)$ relative to the pre-shock benchmark, reflecting the larger contribution rate required for any given retirement inflow. Policy costs around the status quo (s_0, f_0) are measured by $C(f, s)$. The ellipse with the extended height illustrates scenario B where f is costly therefore $\phi_f > \phi_s$, yielding a tall-narrow level set. The ellipse with extended width represents scenario C with costly s-policy thus $\phi_f < \phi_s$, producing a wide-flat set. In each scenario the optimal stationary policy set (f', s') is the tangency between the iso-cost set and the post-shock frontier marked with points. The geometry translates the optimization problem

$$\min_{f \in (0, n+m)} C\left(\alpha \frac{f}{n+m-f}, f\right)$$

into a slope-matching condition at the solution. The frontier's marginal rate of substitution equals the iso-cost slope

$$\alpha \frac{n+m}{(n+m-f')^2} = -\frac{\phi_f}{\phi_s} \frac{f'-f_0}{s'-s_0}; \; s(f') = \alpha \frac{f'}{n+m-f'}.$$

Because the frontier is strictly increasing and convex, and $C(.)$ is strictly convex, the tangency is unique and interior with $(f'-f_0)(s'-s_0) < 0$. The planner tightens retirement entry $f' < f_0$ respectively increases the pensionable age and accepts a higher long-run contribution rate $s' > s_0$, sharing the burden across policy instruments. When f is relatively expensive (scenario B $\phi_f^B / \phi_s^B = 100$) the ellipse is narrow and the tangency slides up the frontier toward a solution closer to f_0 with a larger s'. Conversely, when s is expensive (scenario C $\phi_s^C / \phi_f^C = 100$), the ellipse is flat and the tangency moves left, delivering more adjustment in f and a smaller s' closer to s_0. Figure 6 precisely shows this order based on our calibration. The scenario B optimum is near the vertical line through f_0 with relatively high s, while the scenario C optimum shifts horizontally toward lower f and trims the required s. That the two points lie close to each other in this calibration is informative rather than puzzling: over the displayed range the frontier is well approximated locally by a near-linear segment with slope $s'(f') \approx \alpha/(n+m)$, so a moderate compliance shock rotates the iso-cost schedule without overturning the underlying trade-off, yielding a stable ranking of instruments while still implying a quantitatively meaningful re-optimization close to economic reality.

Economically, the weights ϕ_s and ϕ_f summarize the planner's marginal costs. ϕ_f captures the social and political costs of raising the pensionable age, while ϕ_s captures the marginal excess burden of payroll taxation. The post-shock frontier pins down what is feasible after the compliance shock. The iso-curve selects what is politically desirable. Under any admissible weights the mixed policy dominates the pure corners since strict convexity implies $C_{mix} < \min\{C_{s-only}, C_{f-only}\}$. Dynamic implementation is immediate: on impact, set $s_t = \bar{\rho}d_t/c_1$ to prevent a fall in ρ_t. Legislate $f_t = f'$ permanently as d_t falls according to $d_{t+1} = (d_t + f')/(1+n+m-f')$ the contribution rate drifts monotonically to s' with half-life $\ln 2 / \ln(1/D_d)$ where $D_d = 1/(1+n+m-f')$. The figure's two scenario tangencies therefore map one-for-one to two distinct transitional paths and long-run payroll wedges, providing a transparent, quantitative bridge from normative weights to implementable policy. Under the assumption of budget neutrality (e.g. stable replacement rate) s needs to increase in any mixed scenario but due to lagged demographic easing through f, the contribution rate declines again over time and can reach the s_0 state again ceteris paribus.

5. Discussion and policy implications

The central analytical result of the model is that a compliance shock must be understood as a revenue-side disturbance that creates an immediate wedge between the statutory contribution rules and actual collections. This matters because it changes the timing logic of pension reform. If the shock is demographic, then instruments such as increases are natural responses because they directly affect the contribution-retiree balance. If the shock is instead fiscal and originates in compliance, then the first-order problem is not long-run demography, but short-run budget stabilisation. The model therefore makes a sharp distinction between impact stabilisation and demographic repair. The presented results show that the contribution rate policy is the instrument capable of delivering immediate stabilisation, whereas retirement-age policy operates only gradually through the dependency ratio.

This distinction leads to a first policy implication: Pension systems should not rely on a single instrument when responding to compliance-driven deficits. A contributions-only response is mechanically capable of preserving the replacement rate on impact, but it may be economically and politically costly because it increases the labour-tax wedge and may itself worsen future compliance. An age-only response avoids an immediate increase in payroll burdens, but it cannot prevent an impact decline in pension generosity because demography cannot jump. Therefore, the most important normative conclusion of the model is the superiority of a mixed strategy. Under convex policy adjustment costs, the least-cost long-run solution is generally an interior combination of higher contributions and tighter retirement policy rather than an extreme corner. This result is one of the strongest contributions to the paper, because it provides an explicit analytical rationale for the often-invoked but rarely formalised recommendation to "combine instruments."

A second policy implication follows from the model feasibility results. The model shows that contribution caps can make pure contribution-based stabilisation infeasible, especially when the demographic burden rises further after the shock. This means that feasibility must be assessed dynamically, not just at the moment of impact. In practical terms, pension authorities should evaluate whether targeted benefit protection remains feasible under the worst projected post-shock dependency ratio, not merely under current demographic conditions. This is a valuable refinement of the usual policy discussion, which often treats contribution limits as an ex post political constraint instead of an integral part of the ex ante optimisation problem.

A third implication concerns institutional design. The policy note argues that compliance risk should be internalised in the adjustment architecture of the pension system itself rather than addressed ad hoc after deficits emerge. This argument is strongly supported by the European Commission's 2024 Ageing Report, which notes that only a limited number of EU Member States operate automatic balancing mechanisms that curb pension indexation when deficits arise. The model gives a deeper justification for such mechanisms: where compliance shocks are possible, prespecified rules can convert unexpected revenue shortfalls into predictable, state-contingent corrections. This matters not only for fiscal stabilisation but also for credibility. If contributors know in advance how the system will respond to revenue losses, the scope for credibility erosion, political surprise, and fairness-based compliance deterioration may be reduced.

The discussion also points to an important behavioural interpretation. The four channels summarised in the literature review - salience, credibility, political instability, and distributional fairness, suggest that policy responses are not neutral with respect to future compliance. A sharp payroll response may raise salience and encourage under-reporting; a sequence of ad hoc reforms may damage credibility; repeated reversals may intensify political instability; and burden-shifting across cohorts may trigger fairness objections. In that sense, the model's call for a transparent mixed strategy is not only a result of minimizing direct adjustment costs, but also about minimizing the risk of second-round compliance deterioration. This is where the present paper most clearly extends the earlier sustainability work by Rotschedl (2015) and Rotschedl et al. (2024): it adds a dynamic fiscal channel through which institutional design and reform sequencing influence the effective contribution base itself.

Beyond these channels, recent research suggests that individual and institutional decision making in uncertain environments is often shaped by ambivalence, understood as the coexistence of conflicting motivations and expectations. Ambivalence may not only generate inconsistency, but can also function as a strategic driver that affects adaptability and resilience under changing conditions Petru et al. (2025). In the context of pension systems, such ambivalence can influence contributors' willingness to comply with statutory obligations, particularly when expectations about future benefits are uncertain or contradictory. At the same time, broader macroeconomic conditions, such as inflation dynamics, can further interact with compliance behaviour by altering real expectations about future pension values and the perceived stability of the system. Empirical evidence from V4 countries indicates that inflationary pressures can significantly affect economic decision making and asset-related expectations (Crnadak et al., 2025), which may translate into weakened trust in long-term public schemes such as PAYG pensions. Taken together, these insights suggest that compliance should not be understood solely as a function of policy parameters, but also as an outcome of interacting behavioural and macroeconomic factors, reinforcing the need to extend standard PAYG models to incorporate uncertainty, expectations, and institutional context.

At the same time, the model has limitations that should be acknowledged. Most importantly, the compliance shock is modelled as exogenous and permanent, which is analytically useful because it isolates the fiscal mechanics of the adjustment problem. However, much of the related literature suggests that compliance is itself responsive to contribution rates, transparency, reform credibility, and institutional trust. Therefore, a natural extension would be to endogenize compliance and allow policy choices today to feed back into future effective collections. This would likely reinforce, not weaken, the main normative conclusion of the paper: If aggressive one-instrument responses worsen compliance, then balanced and predictable policy mixes become even more valuable.

Conclusion

This paper identifies a source of instability in PAYG pension systems that is distinct from conventional demographic pressures. While the standard analysis emphasizes ageing, longevity, and labour force dynamics, the present framework shows that a decrease in effective contribution compliance constitutes a separate revenue-side disturbance. Because the number of contributors and retirees remains unchanged on impact, such a shock reduces pension revenue immediately without affecting the dependency ratio. Compliance deterioration should therefore be understood, in the first instance, as a fiscal shock rather than a demographic one. This distinction has direct implications for policy design. In the absence of adjustment, the generosity must fall on impact. A response based solely on retirement age tightening cannot prevent this initial decline since its effects materialize only gradually through the evolution of the dependency ratio. Immediate stabilisation instead requires a contribution-side response. At the same time, once adjustment costs and statutory constraints are taken into account, the optimal long-run response generally involves a combination of contribution increases and retirement age tightening rather than exclusive reliance on either instrument. The policy frontier derived in the paper makes this trade-off explicit and characterises the feasible policy combinations consistent with a given target level of generosity.

The analysis also shows that the feasibility of stabilisation depends not only on the magnitude of the initial compliance loss, but also on the future demographic burden the system must absorb. In particular, contribution-rate ceilings may render contribution-only stabilisation infeasible, especially when the dependency ratio continues to rise. Under such conditions, the retirement age adjustment becomes fiscally necessary rather than merely optional. More broadly, the results indicate that even moderate compliance losses can generate substantial adjustment requirements in otherwise viable PAYG systems.

More generally, the paper links the fiscal mechanics of pension finance to the institutional and behavioural determinants of compliance. Existing work has emphasized the roles of trust, salience, credibility, political risk, and perceived fairness in shaping contribution behaviour. By incorporating compliance directly into the PAYG budget identity, the present framework provides a tractable way to analyse how such factors translate into pension system vulnerability. The central policy implication is that revenue-side compliance risk should be incorporated into pension system design ex ante rather than addressed through discretionary crisis management. In this respect, automatic balancing mechanisms appear particularly relevant, as they can convert revenue shortfalls into predictable, rule-based adjustments, and thereby strengthen the resilience of PAYG systems to both demographic and institutional sources of stress.

References

Ayuso, M., et al. (2021). Automatic indexation of the pension age to life expectancy: When policy design matters. *Risks, 9*(5), Article 96. https://doi.org/10.3390/risks9050096

Bailey, C., Turner, J. (1997). Contribution evasion and social security: *Causes and remedies*. Oficina de la OIT. https://www.issa.int/html/pdf/jeru98/theme3/3-4d.pdf

Bednarczyk, T. H., Szymańska, A., Ostrowska-Dankiewicz, A., & Silva, P. (2023). Life insurance with insurance capital funds as a form retirement savings: Determinants for the self-employed. *Journal of International Studies*, 16(3), 127-143. https://doi.org/10.14254/2071-8330.2023/16-3/7

Besley, T., Prat, A. (2005). Credible pensions. *Fiscal Studies, 26*(1), 119–135. https://doi.org/10.1111/j.1475-5890.2005.00006.x

Blakemore, A. E. et al. (1996). Employer tax evasion in the unemployment insurance program. *Journal of Labor Economics, 14*(2), 210–230. https://doi.org/10.1086/209809

Brycz, M., & Brycz, H. (2025). The relation between personal level of metacognitive self and attitudes towards pension. *Economics and Sociology, 18*(3), 175-183. https://doi.org/10.14254/2071-789X.2025/18-3/10

Brycz, M., Biernat, M., Timiras, L. C., Nichifor, B., & Zait, L. (2024). Expected inheritance and pension attitudes among young people in EU post-communist vs. Anglosphere countries. *Journal of International Studies, 17*(3), 244-257. https://doi.org/10.14254/2071-8330.2024/17-3/13

Choi, J. (2009). *Pension schemes for the self-employed in OECD countries*. OECD. https://www.oecd.org/en/publications/pension-schemes-for-the-self-employed-in-oecd-countries_224535827846.html

Crnadak, O. et al. (2025). Impact of inflation on residential property prices in the V4 countries. *Journal of European Real Estate Research*. Advance online publication. https://doi.org/10.1108/JERER-04-2024-0023

Dankiewicz, R., Balawejder, B., Chudy-Laskowska K., & Britchenko I. (2022). Impact factors and structural analysis of the state's financial security. *Journal of International Studies*, 15(4), 80-92. https://doi.org/10.14254/2071-8330.2022/15-4/5

Dick, T. (2025). The Decision to Save: Extensive and (Subsidized) Intensive Margin on Household Participation in Germany's Third-Pillar Pension. *Proceedings of the International Conference on Economics, Finance & Business, Prague.* https://doi.org/10.20472/EFC.2025.027.001

European Commission. (2024). *2024 Ageing Report: Economic and budgetary projections for the EU Member States (2022–2070)*. Publications Office of the European Union. https://doi.org/10.2765/022983

Iturbe-Ormaetxe, I. (2015). Salience of social security contributions and employment. *International Tax and Public Finance, 22*(5), 741–759. https://doi.org/10.1007/s10797-014-9322-3

Jasova, E., Kaderabkova, B. (2022). The effectiveness of government measures in the first wave of Covid-19 pandemic. *International Journal of Economic Sciences*. 2022, 11(1), 19–36. https://doi.org/10.52950/ES.2022.11.1.002

Kaderabkova, B., Jasova, E. (2019). Development of real unit wage costs on the macro- and mezo-level of the Czech Republic. *International Journal of Economic Sciences*. 2019, vol. 8, no. 2, pp. 45–59. https://doi.org/10.20472/ES.2019.8.2.004

Kangas, O. et al. (2022). Information and legitimacy: Results from an experimental survey on attitudes to the 2017 pension reform in Finland. *Journal of Pension Economics & Finance, 21*(3), 359–374. https://doi.org/10.1017/S1474747220000396

Kumler, T. et al. (2020). Enlisting employees in improving payroll-tax compliance: Evidence from Mexico. *The Review of Economics and Statistics, 102*(5), 881–896. https://doi.org/10.1162/rest_a_00907

Lansky, J. et al. (2022). *Mathematical modelling of qualitative system development*. Mathematics, 10(15), 1–23. https://doi.org/10.3390/math10152752

Li, X. et al. (2020). Missing social security contributions: The role of contribution rate and corporate income tax rate. *International Tax and Public Finance, 27*(6), 1453–1484. https://doi.org/10.1007/s10797-020-09613-6

Leroux, M.-L. et al. (2019). The political economy of contributive pensions in developing countries. *Journal of Public Economic Theory, 21*(2), 262–275. https://doi.org/10.1016/j.ejpoleco.2019.01.002

Mangan, M. et al. (2025). Broken promises? Trust and pension savings in turbulent times. *Work, Aging and Retirement, 11*(2), 139–148. https://doi.org/10.1093/workar/waae007

McGillivray, W. (2001). Contribution evasion: Implications for social security pension schemes. *International Social Security Review, 54*(4), 3–22. https://doi.org/10.1111/1468-246X.t01-1-00102

McHale, J. (2001). The risk of social security benefit-rule changes: Some international evidence. In J. Y. Campbell & M. Feldstein (Eds.), *Risk aspects of investment-based social security reform* (pp. 247–290). University of Chicago Press. https://www.nber.org/books-and-chapters/risk-aspects-investment-based-social-security-reform/risk-social-security-benefit-rule-changes-some-international-evidence

Neugebauer, J., & Vokoun, M. (2024). Economic and Political Dynamics Of Globalization: A Review Of Continuity And Change In Research Focus. *International Journal of Economic Sciences*, *13*(1), 30-57. https://doi.org/10.52950/ES.2024.13.1.003

Petrů, N., & Crnadak, O. (2025). Ambivalence as a strategic driver: A longitudinal SWOT analysis of family business resilience (2015–2024). *Economy: Strategy and Practice, 20*(4), 77–96. https://doi.org/10.51176/1997-9967-2025-4-77-96

Roubal, O. (2023). Consumer Culture and Abundance of Choices: Having More, Feeling Blue. In *A New Era of Consumer Behavior-In and Beyond the Pandemic*. IntechOpen. https://doi.org/10.5772/intechopen.105607.

Rotschedl, J. (2015). Selected factors affecting the sustainability of the PAYG pension system. *Procedia Economics and Finance, 30*, 742–750. https://doi.org/10.1016/S2212-5671(15)01323-4

Rotschedl, J. et al. (2024). The sustainability of PAYG pension schemes: A comparative analysis (1993–2023). *European Journal of Interdisciplinary Studies, 16*(2), 25–51. https://doi.org/10.24818/ejis.2024.10

Samuelson, P. A. (1958). An exact consumption-loan model of interest with or without the social contrivance of money. *Journal of Political Economy, 66*(6), 467–482. https://doi.org/10.1086/258100

Schneider, F., & Boockmann, B. (2023). Die Größe der Schattenwirtschaft – Methodik und Berechnungen für das Jahr 2023. Johannes Kepler University Linz and Institute for Applied Economic Research (IAW) at the University of Tübingen.

Shoven, J. B., & Slavov, S. N. (2006). *Political risk versus market risk in Social Security* (NBER Working Paper No. 12135). National Bureau of Economic Research. https://doi.org/10.3386/w12135

Streimikiene, D. (2025). Ageing society in Baltic States: A comparative study. *Journal of International Studies, 18*(2), 108-119. https://doi.org/10.14254/2071-8330.2025/18- 2/6

Tremblay, M., & Vézina, H. (2000): New estimates of intergenerational time intervals for the calculation of age and origins of mutations. *American journal of human genetics* 66 (2), S. 651–658. https://doi.org/10.1086/302770

van Dalen, H. P., & Henkens, K. (2022). Trust and distrust in pension providers in times of decline and reform: Analysis of survey data 2004–2021. *De Economist, 170*(4), 401–433. https://doi.org/10.1007/s10645-022-09411-x

Yurchyk, H., Mishchuk, H., Bilan, S., & Scare, M. (2024). Social expenditure multiplier: Assessment of economic effect and optimal values. *Economics and Sociology*, *17*(1), 182-195. https://doi.org/10.14254/2071-789X.2024/17-1/12

Appendix

A. Demography: one-state map, fixed point, contraction

Stock-flow identities. Over period t,

$$N_{t+1} = (1+n)N_t - f_t N_t = (1+n-f_t)N_t \quad (A.1)$$
$$R_{t+1} = (1-m)R_t + f_t N_t.$$

Exact ratio dynamics**.** Dividing the two identities in (A.1) by N_t and R_t,

$$d_{t+1} = \frac{R_{t+1}}{N_{t+1}} = \frac{(1-m)R_t + f_t N_t}{(1+n-f_t)N_t} = \frac{(1-m)d_t + f_t}{1+n-f_t}. \quad (A.2)$$

Equation (A.2) is defined for $n > -1$ and $f_t < 1+n$ which holds since $f_t \leq n+m < 1+n$ under our calibration.

For notional symmetry we divide numerator and denominator in (A.2) by $(1-m)$ and define the renormalized rates as

$$\tilde{f}_t \equiv \frac{f_t}{1-m}; \quad \tilde{n}_t \equiv \frac{n+m}{1-m}.$$

Then

$$d_{t+1} = \frac{d_t + \tilde{f}_t}{1+\tilde{n}_t - \tilde{f}_t} \quad (A.3).$$

Relabeling $(\tilde{f}_t, \tilde{n}_t)$ as (f, n) yields the compact representation used in the main text,

$$d_{t+1} = D(d_t, f_t) = \frac{(1-m)d_t + f_t}{1+n-f_t}. \quad (A.4)$$

Equations (A.2) and (A.4) are algebraically equivalent under this change of units; for annual magnitudes with small m, they coincide to first order.

Fixed point. For a constant retirement inflow $f \in [0, n+m)$, a steady state satisfies $d = D(d, f)$. Using either (A.2) or (A.4),

$$d^*(f) = \frac{f}{n+m-f}. \quad (A.5)$$

Contraction and global stability. The map $D(d, f)$ is affine in d. From (A.4),

$$\frac{\partial D}{\partial d}(d, f) = \frac{1-m}{1+n-f} \equiv \lambda(f) \in (0,1). \quad (A.6)$$

Hence $D(d, f)$ is a global contraction on $[0, \infty)$ with modulus $\lambda(f)$. By the contraction mapping theorem, the fixed point in (A.5) is unique and

$$d_t - d^*(f) = \lambda(f)^t \big(d_0 - d^*(f)\big), \text{ for all } t \geq 0, \quad (A.7)$$

which implies monotone convergence with no overshooting.

B. Proof of equation (9) (period budget mechanics and the impact of a compliance shock)

The period budget identity gives $\rho_t = s_t c_t / d_t$. With $s_\tau = s_0$ and d_τ predetermined at the shock date,

$$\rho_{\tau^+} - \rho_{\tau^-} = \frac{s_0}{d_\tau}(c_1 - c_0) = \rho_{\tau^-}\left(\frac{c_1}{c_0} - 1\right), \quad (B.1)$$

hence $\rho_{\tau^+} = \rho_{\tau^-}(c_1/c_0)$. Differentiating $\rho(sc, d) = sc/d$ yields

$$\frac{\partial \rho}{\partial c} = \frac{s}{d}, \qquad \frac{\partial \rho}{\partial d} = -\frac{sc}{d^2}, \quad (B.2)$$

so the log-elasticities are $\varepsilon_{\rho,c} = \left(\frac{\partial \rho}{\partial c}\right)\left(\frac{c}{\rho}\right) = 1$ and $\varepsilon_{\rho,d} = \left(\frac{\partial \rho}{\partial d}\right)\left(\frac{d}{\rho}\right) = -1$. In the linear reparameterization $x \equiv 1/d$, the budget set is $\rho = (sc)x$. A compliance shock rotates the ray by changing its slope from sc_0 to sc_1.

C. Proof, convexity and KKT conditions for the mixed policy case in chapter 2.3

Define the one-dimensional objectives $J(f) \equiv C\big(f, s(f)\big)$ on $[f_{min}, f_{max}]$. Since

$$s'(f) = \alpha \frac{n+m}{(n+m-f)^2} > 0, \quad s''(f) = 2\alpha \frac{n+m}{(n+m-f)^3} > 0, \quad (C.1)$$

we have

$$J'(f) = \phi_f(f - f_0) + \phi_s(s(f) - s_0)s'(f), \quad (C.2)$$

$$J''(f) = \phi_f + \phi_s\left[\big(s'(f)\big)^2 + (s(f) - s_0)s''(f)\right] > \phi_f > 0. \quad (C.3)$$

Thus J is strictly convex on $[f_{min}, f_{max}]$. On a nonempty compact interval, strict convexity implies uniqueness and with continuous objective function existence of a global minimizer f^*. If $f_{min} < f^* < f_{max}$, the necessary and sufficient first-order condition $J'(f^*) = 0$ holds, yielding

$$\phi_f(f^* - f_0) + \phi_s(s(f^*) - s_0)s(f^*) = 0, \quad (C.4)$$

which can be rearranged into the tangency form

$$s(f^*) = -\frac{\phi_f}{\phi_s}\frac{f^* - f_0}{s(f^*) - s_0}. \quad (C.5)$$

Since $s(f^*) > 0$, interior optimality requires $(f^* - f_0)(s(f^*) - s_0) < 0$, given $s(f_0) > s_0$ after a compliance loss, we must have $f^* < f_0$ and $s(f^*) > s_0$.

If f^* lies on a bound, the KKT conditions are

$$J'(f^*) + \mu_+ - \mu_- = 0, \quad \mu_+ \geq 0, \ \mu_- \geq 0, \quad \mu_+(f^* - f_{max}) = 0,$$
$$\mu_-(f_{min} - f^*) = 0.$$

If statutory bounds on s apply, translate them into bounds on f using the inverse frontier $f(s) = \frac{(n+m)s}{\alpha+s}$ and intersect with $[f_{min}, f_{max}]$.

D. Proof of dynamic targeting with feedback, no overshooting and half life in section 3.3

With $c_t = c_1$ and the feedback rule $s_t = \bar{\rho} d_t / c_1$,

$$\rho_t = \frac{s_t c_1}{d_t} = \frac{\left(\frac{\bar{\rho} d_t}{c_1}\right) c_1}{d_t} = \bar{\rho}, \quad (D.1)$$

so the replacement rate is on target for all $t \geq \tau$. Holding $f_t = f'$permanently the one state law of motion gives

$$d_{t+1} = D(d_t, f') = \frac{(1-m)d_t + f'}{1+n-f'} \quad (D.2)$$

The derivate in d is

$$D(d, f') = \frac{\partial G}{\partial d} = \frac{1-m}{1+n-f'} \equiv \lambda \in (0,1), \quad (D.3)$$

so $D(d, f')$ is a contraction. The unique fixed point is through equation (A.5) and geometric convergence follows

$$d_t - d^{**} = \lambda^{t-\tau}(d_\tau - d^{**}), \ \ t \geq \tau. \quad (D.4)$$

Because $\lambda \in (0,1)$, convergence is monotone, no overshooting. For the payroll sequence,

$$s_t - s_\infty = \frac{\bar{\rho}}{c_1}(d_t - d^{**}) = \frac{\bar{\rho}}{c_1}\lambda^{t-\tau}(d_\tau - d^{**}), \quad (D.5)$$

so $s_t \to s_\infty = \bar{\rho} d^{**}/c_1$ at the same rate λ. The half-life $t_{1/2}$ solves $\lambda^{t_{1/2}} = \frac{1}{2}$, hence

$$t_{1/2} = \frac{\ln 2}{-\ln(\lambda)} = \frac{\ln 2}{\ln(1/\lambda)} = \frac{\ln 2}{\ln(1+n+m-f')}. \quad (D.6)$$

E. Dynamic optimization: Lagrangian, Euler equation, and KKT conditions

We solve

$$\min_{\{f_t\}} \sum_{t=\tau}^{\infty} \beta^{t-\tau}\left[\frac{\phi_s}{2}\left(\frac{\bar{\rho}}{c_1}d_t - s_0\right)^2 + \frac{\phi_f}{2}(f_t - f_0)^2\right]$$

subject to

$$d_{t+1} = D(d_t, f_t) \equiv \frac{(1-m)d_t + f_t}{1+n-f_t}, \qquad f_t \in [f_{min}, f_{max}] \subset (0, 1+n), \qquad d_\tau \text{ given.}$$

Let

$$\mathcal{L} = \sum_{t=\tau}^{\infty} \beta^{t-\tau}\left\{\frac{\phi_s}{2}\left(\frac{\bar{\rho}}{c_1}d_t - s_0\right)^2 + \frac{\phi_f}{2}(f_t - f_0)^2 + \mu_{t+1}(D(d_t, f_t) - d_{t+1})\right\}. \quad (E.1)$$

The derivatives of the transition map are

$$D_d(d_t, f_t) = \frac{\partial D}{\partial d} = \frac{1-m}{1+n-f_t}, \qquad D_f(d_t, f_t) = \frac{\partial D}{\partial f} = \frac{1+n+(1-m)d_t}{(1+n-f_t)^2}. \quad (E.2)$$

FOCs:

$$\frac{\partial \mathcal{L}}{\partial f_t} = \beta^{t-\tau}[\phi_f(f_t - f_0) + \mu_{t+1}D_f(d_t, f_t)] = 0$$

$$\Rightarrow \phi_f(f_t - f_0) + \mu_{t+1}\frac{1+n+(1-m)d_t}{(1+n-f_t)^2} = 0 \quad (E.3)$$

$$\frac{\partial \mathcal{L}}{\partial d_t} = \beta^{t-\tau}\left[\phi_s\left(\frac{\bar{\rho}}{c_1}d_t - s_0\right)\frac{\bar{\rho}}{c_1} + \mu_{t+1}D_d(d_t, f_t)\right] = 0$$

$$\Rightarrow \mu_t = \beta\left[\phi_s\left(\frac{\bar{\rho}}{c_1}d_t - s_0\right)\frac{\bar{\rho}}{c_1} + \mu_{t+1}\frac{1-m}{1+n-f_t}\right]. \quad (E.4)$$

With $\beta \in (0,1), f_t < 1+n$, and bounded d_t, a standard transversality condition holds, e.g.

$$\lim_{T\to\infty} \beta^{T-\tau}\mu_{T+1}d_{T+1} = 0.$$

KKT at bounds: If f_t hits a bound, introduce multipliers $\nu_t^+ \geq 0$ for $f_t - f_{max} \leq 0$ and $\nu_t^- \geq 0$ for $f_{min} - f_t \leq 0$. The control FOC becomes

$$\phi_f(f_t - f_0) + \mu_{t+1}D_f(d_t, f_t) + \nu_t^+ - \nu_t^- = 0, \qquad \nu_t^+(f_t - f_{max}) = 0,$$
$$\nu_t^-(f_{min} - f_t) = 0,$$

the costate and state equations remain unchanged.

If the optimal policy is time-invariant $f_t \equiv f^*$ the steady-state limit of the system reproduces the static slope-matching condition between the iso-cost and the post-shock frontier, confirming consistency between the dynamic and steady-state analyses.